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Theoretical estimation of punch velocities and displacements of single-punch and rotary tablet machines

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The speed of travel of punches during compaction by a Manesty F3 single punch and D3B Rotary punch tablet machine has been derived from machine dimensions, normal operating speeds and by consideration of the consolidation of a theoretical compact. The analysis may also be used for machines with other dimensions, operating at different speeds with other materials, but would require modification if the punch head design on the rotary machine differed significantly. Punch speeds at the beginning of the compression cycle were similar for the two types of machines, namely 10·36 and 10·24 cm s^{-1} for the single and rotary machines. The time to reach maximum compression and the total time of contact between punches and powder for the single punch machine was 0·1 s for a compaction force of approximately 40 KN. For the rotary machine operating at approximately the same force, these two parameters were found to be 0·052 and 0·083 s respectively. The additional contact time is associated with a period during which there is no vertical movement of the punch, providing a 'dwell' time of 0·0314 s when the powder is held at a constant volume.

Pharmaceutical tablets are usually produced by either single-sided compaction on a single-punch machine, or by double-sided compaction on a rotary machine. In recent years these two types of machines have tended to become segregated in that the former is used almost exclusively for development and formulation studies, where only a small amount of a drug may be available, while the latter is used almost exclusively for high volume tablet production. Therefore a formulation would be developed to produce satisfactory tablets on a single-punch machine and then be transferred to a rotary machine in a 'scale-up' operation, with the expectation that it would produce satisfactory tablets. However, while the compaction process is generally similar on both types of machine, the tablets produced by a rotary machine may prove to be inferior to those produced on a single punch machine. The speed at which the punch is travelling during the compaction of a powder will affect the physical properties of the final tablet produced (see e.g. Fell & Newton 1971), as the properties of the powder vary with the rate of compaction (see e.g. Hausner 1947). The differences between tablets produced on the two types of machine may be attributable to differences in their punch speeds during compression. In view of the practical difficulties associated with measurements made on high speed rotary tablet presses, it would be useful to obtain expressions for such punch speeds in terms of the physical parameters of the machine (which can be measured accurately with the machine at rest). This may also allow meaningful comparison to be made of the material compaction properties on the two types of machine. In addition, the analysis could provide the basis for an input into tablet machine simulators that are in use for the evaluation of the process of tablet formation and tablet machine performance (Hunter et al 1970).

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SINGLE-PUNCH TABLET MACHINE

A single-punch machine produces the punch motion by having the punch holder attached to an arm which is carried around on an eccentric cam. This can be demonstrated in simplified form in Fig. 1, where D is the distance of the cam off-set, L the length of the punch arm, O the centre of the driving wheel, and P and J are the pivots. A cycle of events is shown schematically in Fig. 2. The motion of the punch can be analysed by considering the motion of the point J, and all displacements will be measured from the point O.

Considering the system to be as shown in Fig. 2(a) initially, i.e. with the punch fully raised, the distance OJ will be:

$$OJ = L - D$$

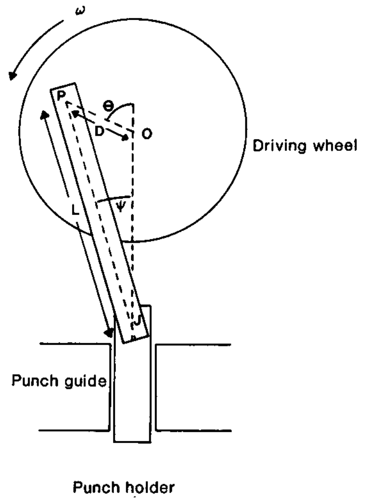

FIG. 1. Diagrammatic representation of single-punch machine.

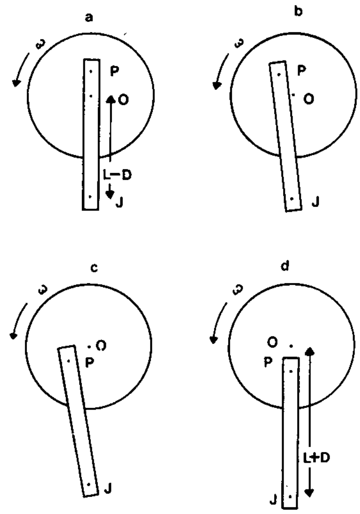

FIG. 2. Diagrammatic representation of the compaction process.

with J vertically below O. As the driving wheel turns to the position shown in Fig. 2(b), the distance OJ becomes:

$$OJ = L.\cos\psi - D.\cos\theta \qquad (1)$$

from Fig. 1. The point J is constrained to move in a vertical plane only, and if the point O is considered as a fixed reference point, the punch speed, V_p, will be given by:

$$V_p = \frac{d(OJ]}{dt} = \frac{d}{dt}[L.\cos\psi - D.\cos\theta] \qquad (2)$$

$$= L.\frac{d}{dt}(\cos\psi) - D.\frac{d}{dt}(\cos\theta) \qquad (3)$$

since both L and D are constants for any given settings on the machine. Now:

$$\frac{d}{dt}(\cos)\,\theta = \frac{d\theta}{dt}\cdot\frac{d}{d\theta}(\cos\theta) = -\sin\theta\cdot\frac{d\theta}{dt} \qquad (4)$$

From Fig. 1:

$$\sin\psi = \frac{D}{L}\cdot\sin\theta \qquad (5)$$

which, since $\cos^2\psi + \sin^2\psi = 1$, may be used to find:

$$\cos\psi = \left(1 - \frac{D^2}{L^2}\cdot\sin^2\theta\right)^{\frac{1}{2}} \qquad (6)$$

Thus:

$$\frac{d}{dt}(\cos\psi) = \frac{d\theta}{dt}\cdot\frac{d}{d\theta}\left[1 - \frac{D^2}{L^2}\cdot\sin^2\theta\right]^{\frac{1}{2}}$$

which, since:

$$\frac{d}{d\theta}\left[1 - \frac{D^2}{L^2}\cdot\sin^2\theta\right]^{\frac{1}{2}} = -\frac{D^2}{L^2}\cdot\left[1 - \frac{D^2}{L^2}\cdot\sin^2\theta\right]^{-\frac{1}{2}}\cdot \sin\theta\cdot\cos\theta \qquad (7)$$

will yield:

$$V_p = D.\sin\theta.\omega.(1 - A.[1 - A^2.\sin^2\theta]^{-\frac{1}{2}}.\cos\theta) \qquad (8)$$

where $A = D/L$, and $\omega = d\theta/dt$, the angular velocity of the driving wheel.

Considering a Manesty 'F' machine, D = 2·5 cm, L = 21 cm, giving A = 0·119, and the angular velocity of the driving wheel

$$\omega = \frac{2.\pi \times \text{No of tablets produced min}^{-1}}{60}\ \text{radians s}^{-1} \qquad (9)$$

Thus, for a machine producing, e.g., 60 tablets $\min^{-1}$, $\omega = 2\pi$ radians s^{-1}.

While the motion of the punch deviates from pure simple harmonic motion over a complete revolution of the driving wheel (Fig. 3a), the only portion of the cycle of interest is that where the punch is in contact with the material being tableted. This portion can be analysed by considering the compaction of a known weight (8 g) of a sample of material with bulk density 1·0402 g cm^{-3} in a 2·5 cm diameter die with flat-faced punches. The initial depth of the die-fill will be:

$$h_I = \frac{M}{\rho_A.\pi.a^2} = \frac{8}{1{\cdot}0402 \times \pi \times (1{\cdot}25)^2}\,\text{cm}$$
$$= 1{\cdot}566\,\text{cm}.$$

The Manesty 'F' machine has a quoted maximum operating load of 4 tons (≈40 kN), which from compression studies was found to produce a compact

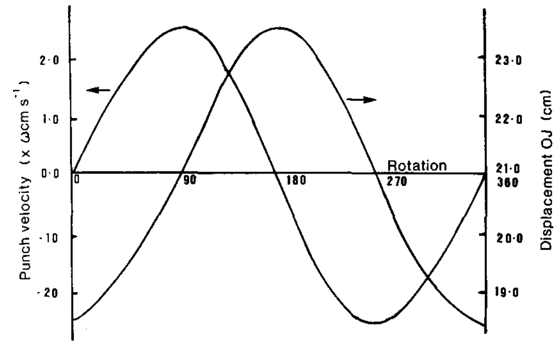

FIG. 3a. Punch velocity and punch displacement vs driving wheel rotation for a Manesty 'F' single punch machine.

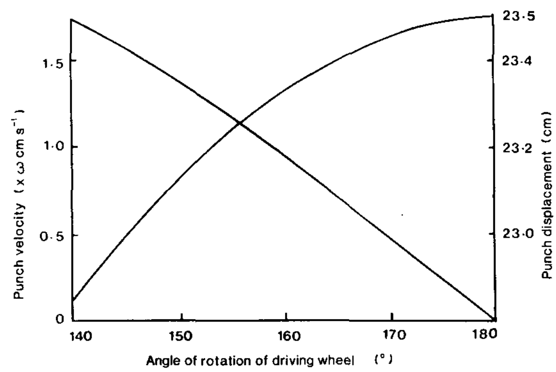

FIG. 3b. Punch velocity and punch displacement vs driving wheel rotation for a Manesty 'F' single punch machine.

of this material of thickness approximately 1·038 cm. Clearly, only the final (1·566–1·038) cm of the punch travel is used in the actual compaction, implying that the driving wheel will have rotated through an angle of θ_c from the position shown in Fig. 2(a) before the punch contacts the powder, which, since $\theta = 180°$ corresponds to the maximum displacement of 23·5 cm, will be given by:

$$23{\cdot}5 - (1{\cdot}566 - 1{\cdot}038) = L. \left[1 - \frac{D^2}{L^2}.\sin^2\theta_c\right]^{\frac{1}{2}} - D.\cos\theta_c \quad (10)$$

i.e. $\theta_c = 144°$.

This is the only portion of the rotation of importance, and is shown expanded in Fig. 3b, which shows that the punch decelerates from approximately 1·65 ω cm s^{-1} at $\theta = \theta_c$, to zero velocity at $\theta = 180°$, in a nearly linear fashion over a time of C_t, where:

$$C_t = \left(\frac{180 - \theta_c}{360}\right).\frac{\omega}{2\pi}\ s \quad (11)$$

For a tablet machine producing 60 tablets min^{-1} (implying $\omega = 2\pi$ radian s^{-1}), with a 'contact angle' of 144° (as before), this time of contact will be 0·1 s, or one-tenth of the time taken for one complete cycle of the machine. For materials compressed to lower loads, this time will be reduced such that the compaction process occurs over less than 10% of the full machine cycle.

ROTARY TABLET MACHINE

Clearly, the proportion of the machine operating time which is spent in performing individual processes can be considered as non-productive, and will limit the maximum production rates obtainable on that machine. An alternative form of tablet machine, which is designed to overcome this problem, is the rotary machine, where compaction is a continuous process, as opposed to the discrete process of a single-punch machine. Rotary machines reach far higher production rates than single-punch machines.

Compaction on a rotary machine is effected by a pair of punches running between two rollers. The upper roller is generally fixed, the lower, or 'pressure', roller being raised to control the amount of punch travel, and hence compaction pressure. The processes occurring at the lower roller and upper roller are seen by analysis to be similar. The movement of the punch will be dependent upon: (a) the height of the roller above the cam track on which the punch runs; (b) the radius of the roller; and (c) the profile of the punch head. The punch speed will then be determined by this movement, and the angular velocity of the turret, which can be converted to an equivalent linear velocity, V_R. The radius of the roller is easily measured, and the height of the roller above the cam track can either be measured or calculated for various settings of this roller, but the punch head profile is harder to define. Fortunately, the punch head can be divided into two areas; a central 'flat', with a curved periphery. Fig. 4 shows part of the profile of a punch designed for use on a Manesty 'D3B' machine (I. Holland Ltd, Long Eaton), measured using an X–Y travelling microscope. The dotted line shown below the punch profile of Fig. 4 represents the shape of head which will be used for this analysis; it is formed from the arc of a circle of radius 1·2 cm, and a flat portion. The vertical displacement of the punch during the compaction stage can be derived from Fig. 4, which also shows the notation used in the analysis. The radius of the roller is R, and that of the curved section of the punch r_c, while ϕ is the angle between the horizontal

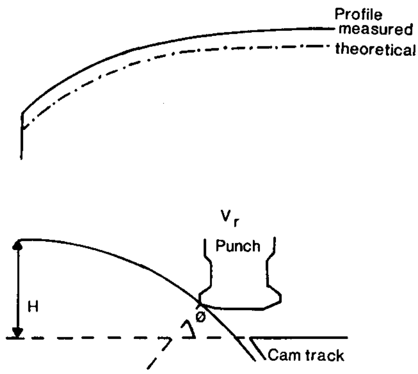

FIG. 4. Punch head profile of a rotary punch (upper) and geometry of the punch passing over the roller (lower).

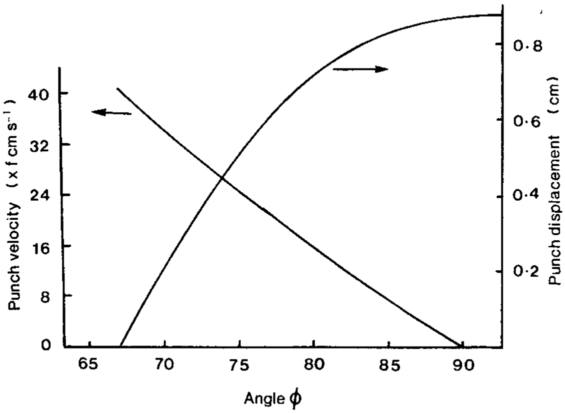

FIG. 5. Punch velocity and punch displacement vs roller/punch angle for a Manesty 'D3B' rotary machine.

cam track and the point of contact of the punch on the roller measured from the centre of the roller.

If the arc of the circle representing the curved portion of the punch head has centre O, then a line joining this point to the centre of the roller will always pass through the point of contact of the roller and punch, except at maximum punch displacement, when the punch flat bears on the roller. This then provides a convenient reference point in the analysis of the punch motion, and the motion of the point O will be analysed.

If the cam track is chosen as the reference level for making calculations of the height of the punch, then the point O will be a distance r_c above this track before the punch reaches the roller. As the punch is lifted by the roller, the height of the point O above the centre of the roller will be:

$$H_R = (R + r_c).\sin\phi \qquad (12)$$

If the height of the roller above the cam track is H (Fig. 4), then the point O will be distance H_p above the track, where:

$$H_p = (R + r_c).\sin\phi - (R - H) \qquad (13)$$

The expression for the punch displacement of equation 13 will hold until $\phi = 90°$, when the punch flat comes into contact with the roller. The height of the punch will then be a constant, until the opposite curved lip contacts the roller, when the punch drops away from the compact.

The velocity of the punch is now governed by the rotation of the turret. When the punch is running on the cam track, it will travel a distance S_H in a small time interval dt, where:

$$S_H = V_R.dt \qquad (14)$$

However, when the punch is running over the roller, this distance can be expressed in terms of the small change of the angle ϕ, $d\phi$. In the same time interval dt, the punch movement horizontally will be:

$$S_H = (R + r_c).\sin\phi.d\phi \qquad (15)$$

During this time, the punch will also have moved a distance S_V vertically, where:

$$S_V = (R + r_c).\cos\phi.d\phi \qquad (16)$$

Clearly, the vertical velocity of the punch, V_V, is now given by the expression:

$$V_V = \frac{S_V}{dt} = \frac{(R + r_c).\cos\phi.d\phi}{dt} \qquad (17)$$

Now, from equation 14:

$$dt = \frac{S_H}{V_R} \qquad (18)$$

which, from equation 15, can be written as:

$$dt = \frac{(R + r_c).\sin\phi.d\phi}{V_R} \qquad (19)$$

giving:

$$V_V = \frac{(R + r_c).\cos\phi.d\phi}{(R + r_c).\sin\phi.d\phi}.V_R$$

$$V_V = \frac{V_R}{\tan\phi} \qquad (20)$$

The velocity V_R is the equivalent linear velocity of the punches in the turret, which is related to the frequency of rotation of the turret, f, and the radius of the circle in which the punches travel, R_p, by the relation:

$$V_R = 2\pi.R_p.f \qquad (21)$$

giving finally:

$$V_V = \frac{2\pi.R_p.f}{\tan\phi} \qquad (22)$$

The analysis of the compaction process on a rotary machine is complicated by the relative movements of the two punches. If one punch travels a considerable distance farther than the other (the upper roller being fixed), the compaction processes will be hard to specify exactly, and the punch motions will have to be considered in stages. However, it is common to assume that the two punches move equal amounts in opposite directions (Rippie & Danielson 1981), and this greatly simplifies any calculations which are to be made.

Considering once again the compaction of an 8 g sample of a model material in a 2·5 cm diameter die with flat-faced punches, as in the previous section, the compact thickness at 40 kN is approximately 1·038 cm. From measurements made of a Manesty 'D3B' machine, the lower edge of the upper roller is approximately 28·5 cm above the lower cam track, and the punches are approximately 13·3 cm long with both upper and lower rollers having a radius of approximately 11·3 cm. Equation 13 gives the height of the lower punch tip from the cam track, H_L, as:

$$H_L = (R + r_c).\sin\phi - (R - H) + (L_p - r_c) \quad (23)$$

where L_p is the overall length of the punch. A similar consideration of the geometry of the upper roller will give an expression for the height of the tip of the upper punch above the lower cam track, H_T:

$$H_T = D_R - L_p + (R + r_c).(1 - \sin\phi) \quad (24)$$

where D_R is the height of the lower edge of the upper roller above the lower cam track.

Clearly, the separation of the punch tips, P_s, is then given by:

$$P_s = H_R - H_L$$
$$= D_R - H - 2L_p + 2(R + r_c).(1 - \sin\phi) \quad (25)$$

Substitution of the measured values in equation 25, and rearrangement, gives:

$$\sin\phi = 1 - \frac{P_s - 28{\cdot}5 + H + 26{\cdot}6}{25} \quad (26)$$

At the point of maximum compression, i.e. $\phi = 90°$, it is required that $P_s = 1{\cdot}038$ cm. Substitution for ϕ and P_s into equation 26 gives $H = 0{\cdot}862$ cm. Thus:

$$\sin\phi = 1 - \frac{P_s - 1{\cdot}038}{25}$$
$$= 1{\cdot}04152 - \frac{P_s}{25} \quad (27)$$

On the assumption that once again the initial depth of the die-fill is 1·566 cm, this bed will be lifted with the lower punch as it runs over the lower roller until the upper punch is forced onto the powder by the upper roller. From the symmetry of the system, the two punches will be running on corresponding portions of the two rollers, and their velocities at any time will be equal but opposite in direction. At the point where compaction commences, $P_s = 1{\cdot}566$ cm, i.e. from equation 27.

$$\sin\phi_c = 1{\cdot}04152 - \frac{1{\cdot}566}{25}$$

giving:

$$\phi_c = 78{\cdot}2° \quad (28)$$

If this machine is assumed to be producing tablets at a rate of $480\,\text{min}^{-1}$, this, with 16 stations, will correspond to a frequency of rotation of the turret, f, of $0{\cdot}5\,\text{s}^{-1}$. The measured radius of the circle on which the centre of the dies runs, R_p, is approximately 15·6 cm, which, from equations 22 and 28, yields the punch velocity at the start of compaction as:

$$V_V = \frac{2\pi.R_p.f}{\tan\phi_c} = 10{\cdot}24\,\text{cm s}^{-1} \quad (29)$$

The value of the lower roller lift, H, required for this purpose will cause the lower punch to strike the roller at a value of ϕ of approximately 67·5°, and the velocity and displacement of the punch from this point to the time when the punch drops away from the compact are shown in Fig. 6. While the punch flat

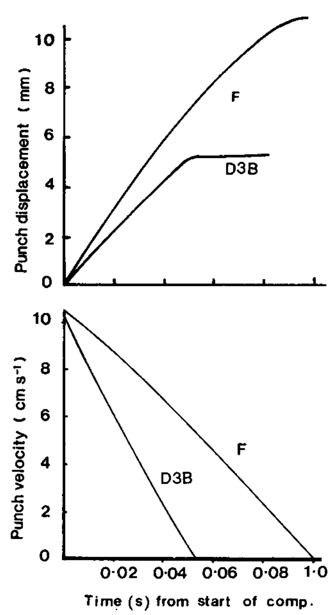

FIG. 6. Punch displacements and velocities vs time.

is running over the roller, the value of ϕ, punch velocity and punch displacement are constant at 90°, $0\,m\,s^{-1}$ and H m respectively. This feature of the punch motion represents another difference between a single-punch and a rotary machine, since for a time t_f the punch displacement is constant, and the pressure is maintained on the powder for this time. From equation 21:

$$t_f = \frac{L_f}{2\pi . R_p . f} \quad (30)$$

where L_f is the width of the flat portion of the punch head. The measurements made on the 'D3B' punch give L_f as approximately 15·4 mm, so that:

$$t_f = \frac{0{\cdot}0157}{f}\,s \quad (31)$$

If the machine is again assumed to be producing 480 tablets min^{-1}, then $f = 0{\cdot}5\,s^{-1}$, and t_f will become 0·0314 s. The total time spent in compacting a sample can now be calculated, since, from equation 28, $\phi_c = 78{\cdot}2°$, implying that the punch travels a horizontal distance of $(R + r_c).\cos(\phi_c)$ during the compression stage, which from equation 21, will take a time t_c, where:

$$t_c = \frac{(R + r_c).\cos\phi_c}{2\pi . R_p . f}\,s \quad (32)$$

$$= 0{\cdot}052\,s.$$

Thus the total contact time of the punches and the powder will be C_t:

$$C_t = t_f + t_c = 0{\cdot}083\,s. \quad (33)$$

COMPARISON OF TABLET MACHINES

The two types of tablet machine may now be compared, and from equation 29, and Fig. 6, it is seen that the punch speeds at the start of compaction of either machine are similar (i.e. $10{\cdot}36\,cm\,s^{-1}$ for a single-punch, and $10{\cdot}24\,cm\,s^{-1}$ for a 'D3B' rotary machine). However, because the powder is subjected to compaction from both sides on a rotary machine, the compression times will be different: 0·1 s on a single-punch machine (eqn 11), and 0·052 s on a 'D3B' machine (eqn 32). While the compression times for a single-punch machine are equal to the contact time between the powder and punches, the 'D3B' machine has an extra 'dwell' time, due to the flat portion of the punch-head, giving a contact time of 0·083 s.

These differences in punch displacements and velocities during a compression cycle are shown in Fig. 6, where displacements and time are calculated from the point where the punch first contacts the powder, i.e. when compression begins, to the point when the punch moves away from the tablet surface. The displacement curves are shown for the length of time over which the punch will be in contact with the material, assuming zero strain relaxation in the compact as the pressure is released. Fig. 6 shows the shorter time to reach the maximum displacement for the rotary machine, which suggests that this machine will be more likely to induce capping or lamination in the compaction of brittle or 'elastic' materials than the slower single punch machine. However, the extended 'dwell time' produced implies that the pressure is applied at the maximum level for a longer time, which may produce plastic flow in any component of the material exhibiting plastic properties, thus absorbing the energy of elastic strain recovery (Doekler & Shotton 1977) before the pressure is released, and this may be sufficient to prevent the excessive strain relaxation in the compact which is considered to be a cause of capping (Hiestand et al 1977). Clearly, if any plastic flow occurs, the value of the pressure on the punch will not be constant, but will decrease, with the compact held at constant volume, with time.

The calculations of punch displacement and velocity made here suggest that the compaction process is different between the two types of tablet press, and that a product which compresses satisfactorily on a single punch machine may not necessarily produce acceptable tablets on a rotary machine, because of the greater elastic strains being induced in the powder bed. In order for the scale-up of a compaction process from a single punch to rotary machine to be successful, it seems likely that the formulation should include a component showing some plastic properties to absorb elastic strain recovery energy while the sample is held under pressure (i.e. during the constant volume dwell time stage of compression), since this will be the period of maximum deformation of the sample (Wells et al 1982).

These calculations also show that while the single-punch machine is performing compaction for approximately 10% of the tableting cycle, each die on a rotary machine is in use for $(0{\cdot}083/2)\,s^{-1}$, i.e. approximately 4% of the cycle for each die. Clearly, the multiplicity of dies on the turret will increase this efficiency, since the compression process can be made continuous, further reducing machine 'dead-time'.

This analysis will be subject to some minor errors due to the assumptions made about the operation of the relevant machines. The calculations have been made on the basis of no deformation of the various components involved (e.g. punches, cams etc.) and that the compaction loads do not exceed the pressure relief settings. Changes in punch movement have been shown to occur in single-punch tablet machines due to the stiffness of the bearings of the machines (Kennerley et al 1981) and no doubt all machines vary in their stiffness because of wear. A further effect will arise from the assumption that the upper punch of a rotary machine follows the profile of the upper roller. In practice, this punch will fall from the upper cam track onto the powder under its own weight before running under the upper roller. The powder bed will thus be subjected to some tapping, and to some pre-compression before the compaction process, leading to a thinner sample than has been assumed in deriving equation 28. The approach to the problem is not restricted to these particular tablet machines, nor are the answers absolute. With suitable, relatively simple, modifications it would be possible to allow for machine variables such as the design and properties of punches, cams, rollers and machine stiffness. These could provide a basis for use in the design of cam and punch profiles.

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Notation

- A ratio D/L for single punch machine
- a radius of tablet die
- C_t time of contact between punch and powder
- D cam offset of a single-punch tablet machine
- D_R height of upper roller of a rotary tablet machine above turret
- f frequency of rotation of a rotary tablet machine
- H height of roller on a rotary tablet machine
- H_I height of fill of powder bed in a tablet die
- H_p, H_R height of punch on a rotary machine
- H_T height of punch tip on a rotary machine
- L length of arm on a single-punch machine, cm
- L_f width of flat on a punch of a rotary tablet machine
- L_P length of punch
- M weight of die-fill
- P_s separation of punch tips
- r_c radius of curvature of the curved portion of head of punch of a rotary tablet machine
- R radius of compression rollers on rotary machine
- R_P radius of circle of dies on the turret of a rotary tablet machine
- S_H rotary punch displacement in horizontal direction
- S_V rotary punch displacement in vertical direction
- t_c compression time
- t_f time during which punch head flat runs over roller on a rotary tablet machine
- V_P punch velocity—single-punch machine
- V_R punch velocity in horizontal direction on a rotary machine
- V_V punch velocity in vertical direction on a rotary machine
- θ angle between centre of drive wheel and punch arm
- ρ_A apparent density of compact
- θ_c value of θ at which compaction commences
- ϕ angle between the horizontal cam track and point of contact on the roller, measured from centre of roller
- ϕ_c value of ϕ at which compaction commences
- ψ angle between punch arm and vertical on single-punch machine

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